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Abstract

We study the optimal mix of direct and indirect taxes in the presence of tax avoidance. It is shown that a linear consumption tax is part of the optimal tax scheme, and so even when the assumptions of the Atkinson-Stiglitz theorem are not satisfied. The reason is that taxing consumption is a way to tax *true* income, whereas income taxation only falls on *reported* income. We also show that, with a weakly separable utility function and linear Engel curves, tax rates should be uniform across goods. With nonlinear Engel curves, which good should be taxed more depends on the comparison of a redistributive and an efficiency effects.

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1 Introduction

The literature on optimal taxation, initiated by the pioneering work of Mirrlees (1971) has for a long time focused on the labor supply response to taxes. This corresponds to what Slemrod (1995) names the *real* response to taxation. However individuals can also react to taxes by manipulating their reported income, keeping labor supply unchanged. This response can be legal (avoidance) or not (evasion).

We focus in this paper on tax avoidance, that is legal means used by individuals to reduce their tax payments. Tax avoidance is a pressing issue in all advanced economies. According to a 2011 report of the UK's economics and finance ministry (Tackling Tax Avoidance), avoidance activities are estimated to represent about 17.5 % of the overall tax gap (42 billion £), which is defined as the difference between taxes owed and taxes paid.

The academic literature on this issue has been growing quickly these last years. Part of it is devoted to the study of the elasticity of taxable income (see the survey by Saez et al. (2011)). This concept, which encompasses both real and substitution responses to taxation, is indeed the relevant one for evaluating tax distortions. There also exist a few papers that concentrate more on the avoidance margin. Slemrod (2001) studies the effect of income taxation in a model where both real (change in labor supply) and avoidance responses are taken into account. He does however adopt a purely positive standpoint and does not determine the optimal level of taxes. Slemrod and Kopczuk (2002) determine the optimal level of avoidance. Contrarily to labor supply responses, avoidance behaviors can be, at least partly, controlled by the government. This has crucial implications for the design of the tax system. If avoidance responses to taxation are large, the best policy would not be to lower tax rates (as suggested by the standard Mirrleesian approach), but instead to broaden the tax base and eliminate avoidance opportunities. Roine (2006) develops a political economy model in which individuals vote on the level of the tax, when the distortion caused by the tax is driven by avoiding behaviors.

None of the previous papers develop an optimal taxation approach. Grochulski (2007) and Casamatta (2011) determine the optimal income tax curve when individuals have access to a costly avoidance technologies. Grochulski (2007) shows that, with a sub-additive cost function, individuals should not avoid taxes at the optimum. In other words, all individuals should declare their true income. Casamatta (2011) considers a convex cost function (that violates sub-additivity). He finds that middle-class individuals should be allowed to avoid taxes. In these two articles, the only instrument at the disposal of the tax administration is the (direct) tax on labor income. In this article, we introduce indirect taxes, assuming that both labor income and consumption goods can be taxed. We moreover take into account both the labor supply and avoidance responses. It should be emphasized that we are not the first ones to take up this issue. Boadway et al. (1994) have developed a very similar model. We however provide an alternative method for solving the problem, hoping to bring additional insights and intuitions, and thus to complement the analysis by Boadway et al. (1994).

The setup considered allows us to contribute to the direct versus indirect tax controversy. This debate originates from the celebrated article by Atkinson and Stiglitz (1976). These latter show that there is no need to tax consumption, as long as the utility function is weakly separable between consumption and leisure. This result is known as the Atkinson-Stiglitz theorem. We show that considering tax avoidance has profound implications on the result. The reason is that, with tax avoidance, *true and declared incomes do not coincide.* As income taxes fall on declared income only, there is thus a scope for indirect taxation, even when the assumptions of the Atkinson-Stiglitz theorem are violated. This follows from the observation that taxing consumption is equivalent to taxing true income and thus allows to increase social welfare with respect to the case where only the (nonlinear) labor income tax is used. The framework analyzed thus provide a rationale for the existence of a direct/indirect tax mix, where both direct and indirect taxes coexist. The reasoning exposed in the previous paragraph holds true with a single consumption good. We also study the case of two consumption goods, tackling the issue of the tax differentiation between goods. We find, as in Boadway et al. (1994), that tax rates should be identical between goods, provided that the utility function is weakly separable between consumption and leisure *and* Engel curves are linear. With nonlinear Engel curves, matters are more complicated. We show that two effects play in opposite directions. Increasing a given tax rate has both a redistributive and an efficiency effects. We show through a simple example that these two effects may go in opposite directions. Even though taxing more luxuries is beneficial from a redistributive standpoint, it also induces more distortions. We cannot thus conclude analytically on the desirability of taxing more luxuries than necessities. Which goods should be taxed the more depends on which of the two effects identified above dominates.

The remainder of this article is organized as follows. The model is presented in section 2. In section 3, we consider a single consumption good. Finally section 4 is devoted to the case of two consumption goods.

2 Model

We consider an economy with three goods: leisure l and two consumption goods. Individuals differ in productivity: $\omega^L < \omega^H$, with respective numbers n^L and n^H. They have the possibility to reduce (legally) their taxable income. We denote y true labor income and $\hat{y}$ taxable labor income. The cost of hiding Δ euros is $\phi(\Delta)$, such that $\phi(0) = 0$, $\phi' > 0$, $\phi'' > 0$.

We write the utility function as follows:

$$\begin{aligned} U^h &= u(c_1^h, c_2^h, l^h) \\ &= u(c_1^h, c_2^h, y^h/\omega^h), \end{aligned}$$

where c_i^h is the consumption of good i by a type h individual and l^h his labor supply. In the next section, we first address the case of an economy with a single consumption good. This helps to clarify how tax avoidance affects the design of the tax structure.

3 A single consumption good

3.1 Optimal income taxation

We assume as a starting point that the government does not tax consumption goods. Tax revenues only come from labor income. The problem is solved in two steps. In the first stage, the planner determines the amount of taxable income $\hat{y}^h$ and the level of the tax T^h for each type of individual. Then individuals choose optimally, in the second stage, their labor supply. We proceed backward and solve first the program of the individuals in the second stage:

$$\begin{aligned} & \max u(c^h, y^h/\omega^h) \\ & \text{st} \\ c^h &= y^h - T^h - \phi(y^h - \hat{y}^h). \end{aligned}$$

The first-order condition is:

$$(1-\phi')u_c^h + \frac{1}{\omega^h}u_l^h = 0.$$

We substitute the budget constraint in the utility function, to obtain the indirect utility function:

$$V^h(\hat{y}^h, T^h) = u(y^h - T^h - \phi(y^h - \hat{y}^h), y^h/\omega^h).$$

Differentiating this function and using the envelop theorem, we get:

$$\frac{\partial V^h}{\partial \hat{y}^h} = \phi'(y^h - \hat{y}^h)u_c^h, \;\; \frac{\partial V^h}{\partial T^h} = -u_c^h.$$

We also write the corresponding expressions for the mimicking individual. When a type H mimics a type L, he gets the utility level:

$$\begin{aligned} \tilde{V}^H &= V^H(\hat{y}^L, T^L) \\ &= u(\tilde{c}^H, \tilde{y}^H/\omega^H) \\ &= u(\tilde{y}^H - T^L - \phi(\tilde{y}^H - \hat{y}^L), \tilde{y}^H/\omega^H), \end{aligned}$$

where $\tilde{y}^H$ is the income level that maximizes the utility of type H individuals when these latter declare $\hat{y}^L$ and pay the income tax T^L. We thus have

$$\frac{\partial \tilde{V}^H}{\partial \hat{y}^L} = \phi'(\tilde{y}^H - \hat{y}^L)\tilde{u}_c^H, \;\; \frac{\partial \tilde{V}^H}{\partial T^L} = -\tilde{u}_c^H.$$

We now address the program of the (utilitarian) social planner in the first stage. It chooses bundles $(T^L, \hat{y}^L)$ and $(T^H, \hat{y}^H)$ in order to solve the following program:

$$\begin{aligned} & \max_{T^h, \hat{y}^h} n^L u(y^L - T^L - \phi(y^L - \hat{y}^L), y^L/\omega^L) + n^H u(y^H - T^H - \phi(y^H - \hat{y}^H), y^H/\omega^H) \\ & \text{st} \\ & n^L T^L + n^H T^H \geq 0 \\ & u(y^H - T^H - \phi(y^H - \hat{y}^H), y^H/\omega^H) \geq u(\tilde{y}^H - T^L - \phi(\tilde{y}^H - \hat{y}^L), \tilde{y}^H/\omega^H). \end{aligned}$$

Denoting μ and λ the Lagrange multipliers of the government budget constraint (GBC) and the incentive constraint respectively, first-order conditions for interior solutions read as:

$$\begin{aligned}
-n^L u_c^L + \mu n^L + \lambda \tilde{u}_c^H &= 0 \\
-n^H u_c^H + \mu n^H - \lambda u_c^H &= 0 \\
n^L \phi'(y^L - \hat{y}^L) u_c^L - \lambda \phi'(\tilde{y}^H - \hat{y}^L) \tilde{u}_c^H &= 0 \\
n^H \phi'(y^H - \hat{y}^H) u_c^H + \lambda \phi'(y^H - \hat{y}^H) u_c^H &= 0.
\end{aligned}$$

It is clear that the last condition cannot be satisfied with equality, as the left-hand side is strictly positive. This implies that $y^H = \hat{y}^H$: *highly productive individuals should not avoid taxes at the second-best optimum.*

As for the types L, we can deduce easily from the third condition that low productivity individuals should be allowed to avoid taxes as soon as the marginal cost of avoiding the first euro, $\phi'(0)$, is low enough. In this case, allowing avoidance at the margin does not hurt much the types L, as the marginal cost $\phi'(0)$ is low. But it allows to relax the incentive constraint, as represented by the term $\lambda \phi'(\tilde{y}^H - \hat{y}^L) \tilde{u}_c^H$. The avoidance cost being convex, it is more costly for the types H to mimic the types L. These results were demonstrated by Casamatta (2011) in a continuous type model with exogenous income. The lesson from this simple two types model is that they extend to the case of endogenous labor supply.

3.2 Tax implementation

When faced with a tax function $T(.)$, a type h individuals solves the following program:

$$\begin{aligned}
&\max_{y^h, \hat{y}^h} u(c^h, y^h/\omega^h) \\
&\text{st} \\
c^h &= y^h - T(\hat{y}^h) - \phi(y^h - \hat{y}^h),
\end{aligned}$$

leading to the first-order condition:

$$T'(\hat{y}^h) = \phi'(y^h - \hat{y}^h).$$

The marginal tax rate is equal to the marginal avoidance cost. This is intuitive. Individuals adjust their reported income up to the point where the save in income tax equals the additional cost of hiding income. This implies that highly productive individuals face a marginal tax rate equal to $\phi'(0)$. The marginal tax rates of the low types is $\phi'(y^L - \hat{y}^L) > \phi'(0)$. As soon as $\phi'(0) \neq 0$, this implies that *marginal tax rates are strictly positive for all individuals.* This contrasts with the conclusion of the standard optimal taxation model, in which the marginal tax rate of the more able is 0 (Stiglitz (1987)).[1]

3.3 Direct vs. indirect taxation

We now assume that consumption can be taxed at a proportional rate t. Confronted to this tax rate, a type h individual determines his optimal consumption level and labor supply to solve:

$$\max_{c^h, y^h} u(c^h, y^h/\omega^h)$$

st

$$(1+t)c^h = y^h - T^h - \phi(y^h - \hat{y}^h).$$

Individually optimal consumption level thus satisfy the first-order condition:

$$\frac{1-\phi'}{1+t} u_c^h = \frac{1}{\omega^h} u_l^h.$$

We substitute the budget constraint in the utility function, to obtain the indirect utility function:

$$\begin{aligned} V^h(t, \hat{y}^h, T^h) &= u(c^h, y^h/\omega^h) \\ &= u(\frac{y^h - T^h - \phi(y^h - \hat{y}^h)}{1+t}, y^h/\omega^h). \end{aligned}$$

Differentiating this function and using the envelop theorem, we get:

$$\frac{\partial V^h}{\partial t} = -\frac{c^h}{1+t} u_c^h, \quad \frac{\partial V^h}{\partial \hat{y}^h} = \frac{\phi'(y^h - \hat{y}^h)}{1+t} u_c^h, \quad \frac{\partial V^h}{\partial T^h} = -\frac{1}{1+t} u_c^h. \tag{1}$$

[1] Boadway et al. (1994) obtain also a zero marginal tax rate at the top. Their result however comes from the fact that they concentrate in their model on the particular case where $\phi'(0) = 0$.

We also write the corresponding expressions for the mimicking individual. When a type H mimics a type L, he gets the utility level:

$$\begin{aligned}\tilde{V}^H &= V^H(t, \hat{y}^L, T^L) \\ &= u(\tilde{c}^H, \tilde{y}^H/\omega^H) \\ &= u(\frac{\tilde{y}^H - T^L - \phi(\tilde{y}^H - \hat{y}^L)}{1+t}, \tilde{y}^H/\omega^H).\end{aligned}$$

We thus have

$$\frac{\partial \tilde{V}^H}{\partial t} = -\frac{\tilde{c}^H}{1+t}\tilde{u}_c^H, \quad \frac{\partial \tilde{V}^H}{\partial \hat{y}^h} = \frac{\phi'(\tilde{y}^H - \hat{y}^L)}{1+t}\tilde{u}_c^H, \quad \frac{\partial \tilde{V}^H}{\partial T^L} = -\frac{1}{1+t}\tilde{u}_c^H. \tag{2}$$

It is straightforward to show that $\tilde{y}^H > y^L$.[2] Consumption being a normal good, this implies $\tilde{c}^H > c^L$.

The social planner chooses the tax rate t and proposes two bundles $(T^L, \hat{y}^L)$ and $(T^H, \hat{y}^H)$. These bundles are designed such that individuals optimally choose the one intended for them. A utilitarian planner then solves the following program:

$$\max_{t, T^h, \hat{y}^h} n^L V^L(t, \hat{y}^L, T^L) + n^H V(t, \hat{y}^H, T^H)$$

st

$$t(n^L c^L + n^H c^H) + n^L T^L + n^H T^H \geq 0$$

$$V^H(t, \hat{y}^H, T^H) \geq V^H(t, \hat{y}^L, T^L).$$

The first-order conditions with respect to t and T^h are:

$$\begin{aligned} n^L\frac{\partial V^L}{\partial t} + n^H\frac{\partial V^H}{\partial t} + \mu(n^L c^L + n^H c^H + t(n^L\frac{\partial c^L}{\partial t} + n^H\frac{\partial c^H}{\partial t})) + \lambda(\frac{\partial V^H}{\partial t} - \frac{\partial \tilde{V}^H}{\partial t}) &= 0 \\ n^L\frac{\partial V^L}{\partial T^L} + \mu(tn^L\frac{\partial c^L}{\partial T^L} + n^L) + \lambda(-\frac{\partial \tilde{V}^H}{\partial T^L}) &= 0 \\ n^H\frac{\partial V^H}{\partial T^H} + \mu(tn^H\frac{\partial c^H}{\partial T^H} + n^H) + \lambda\frac{\partial V^H}{\partial T^H} &= 0. \end{aligned}$$

[2] Evaluate $\partial \tilde{U}^H/\partial \tilde{y}^H$ at $\tilde{y}^H = y^L$. At this point, the type H mimicking individual chooses the same consumption basket. Therefore:

$$\left.\frac{\partial \tilde{U}^H}{\partial \tilde{y}^H}\right|_{\tilde{y}^H = y^L} = \frac{1-\phi'(.)}{1+t}u_c^L - \frac{1}{\omega^H}u_l(c, \frac{y^L}{\omega^H}).$$

Recalling that

$$\frac{\partial U^L}{\partial y^L} = \frac{1-\phi'(.)}{1+t}u_c^L - \frac{1}{\omega^L}u_l(c, \frac{y^L}{\omega^H}) = 0$$

and $\omega^H > \omega^L$, we obtain that $\left.\partial \tilde{U}^H/\partial \tilde{y}^H\right|_{\tilde{y}^H = y^L} > 0$.

We use (1) and (2), as well as the Slutsky relationship:

$$\frac{\partial c^h}{\partial t} = S^h + c^h \frac{\partial c^h}{\partial T^h},$$

where S^h is the Slutsky substitution term. Multiplying the second and third conditions by c^L and c^H respectively and adding up, we obtain:

$$n^L c^L \frac{\partial V^L}{\partial T^L} + \mu(tn^L c^L \frac{\partial c^L}{\partial T^L} + n^L c^L) + \lambda c^L(-\frac{\partial \tilde{V}^H}{\partial T^L}) + n^H c^H \frac{\partial V^H}{\partial T^H} + \mu(tn^H c^H \frac{\partial c^H}{\partial T^H} + n^H c^H) + \lambda c^H \frac{\partial V^H}{\partial T^H} = 0.$$

Substracting:

$$\frac{\partial \mathcal{L}}{\partial t} = \mu t(n^L S^L + n^H S^H) + \lambda \frac{\partial \tilde{V}^H}{\partial T^L}(c^L - \tilde{c}^H).$$

We know from previous analysis that the second term is positive. Increasing t has thus a positive effect on the incentive constraint: as the type H mimicker consumes more of the good than the type L, he suffers more from an increase in the tax and is thus less tempted to mimic. This instrument is different from an increase in T^L, as the same amount would be taken to both the type L and the type H mimicker. With the consumption tax, the mimicker pays more taxes than the mimicked. An increase in t has also a negative effect on the GBC: it induces individuals to *substitute* leisure for consumption and thus decreases fiscal revenues. The optimal tax rate thus strikes a balance between the positive redistributive effect, and the negative efficiency one. Evaluating the above expression at $t = 0$, it is clear that *the optimal tax rate is positive*.

The main message of this section is that, in the presence of tax avoidance, taxing consumption allows to improve social welfare. The conclusion of the Atkinson-Stiglitz theorem, which states that consumption should not be taxed, unless the utility function displays some separability properties, thus does not extend to the avoidance framework. This comes from the fact that only reported labor income can be taxed through the income tax, and not true income. The use of the consumption (which is equal to true income in this static model) tax constitutes then a way to tax true labor income. Without avoidance, the incomes of the mimicker and the mimicked individuals are the same and the consumption tax thus plays no role. Here the mimicker has the same *reported* income but earns a different level of income.

Note that, to obtain our result, we have used first-order conditions for interior solutions of T^L and T^H. In other words, it is desirable to have a mix of direct and indirect taxation at the optimum. As the consumption tax allows to tax true income and thus deters avoidance possibilities, why is it so that the government also relies on direct income taxes? The reason is simply that, contrarily to the consumption tax, the income tax is nonlinear and thus allows to apply distinct marginal tax rates to poor and rich individuals. In other words, the consumption tax is immune to avoidance behaviors but it is also less flexible than the income tax. Should we have allowed for nonlinear indirect taxes, no income taxation would have been necessary.

4 Tax differentiation

We rule out nonlinear consumption taxes and denote t_1 and t_2 the ad-valorem taxes on goods 1 and 2 respectively. Confronted to these tax rates, a type h individual determines his optimal consumption levels and labor supply to solve:

$$\begin{aligned} \max_{c_1^h, c_2^h, y^h} u(c_1^h, c_2^h, y^h/\omega^h) & \\ \text{st} & \\ (1+t_1)c_1^h + (1+t_2)c_2^h = & \quad y^h - T^h - \phi(y^h - \hat{y}^h). \end{aligned}$$

Individually optimal consumption levels thus satisfy the first-order conditions:

$$\frac{u_{c_1}^h}{u_{c_2}^h} = \frac{1+t_1}{1+t_2}.$$

We substitute the budget constraint in the utility function, to obtain the indirect utility function:

$$\begin{aligned} V^h(t_1, t_2, \hat{y}^h, T^h) &= u(c_1^h, c_2^h, y^h/\omega^h) \\ &= u(\frac{y^h - T^h - \phi(y^h - \hat{y}^h) - (1+t_2)c_2^h}{1+t_1}, c_2^h, y^h/\omega^h) \\ &= u(c_1^h, \frac{y^h - T^h - \phi(y^h - \hat{y}^h) - (1+t_1)c_1^h}{1+t_2}, y^h/\omega^h). \end{aligned}$$

Differentiating this function and using the envelop theorem, we get:

$$\frac{\partial V^h}{\partial t_1} = -\frac{c_1^h}{1+t_2}u_{c_2}^h, \ \frac{\partial V^h}{\partial t_2} = -\frac{c_2^h}{1+t_1}u_{c_1}^h = \frac{c_2^h}{c_1^h}\frac{\partial V^h}{\partial t_1}, \ \frac{\partial V^h}{\partial \hat{y}^h} = \frac{\phi'(y^h - \hat{y}^h)}{1+t_1}u_{c_1}^h, \ \frac{\partial V^h}{\partial T^h} = -\frac{1}{1+t_1}u_{c_1}^h. \tag{3}$$

We also write the corresponding expressions for the mimicking individual. When a type H mimics a type L, he gets the utility level:

$$\begin{aligned}\tilde{V}^H &= V^H(t, \hat{y}^L, T^L)\\ &= u(\tilde{c}_1^H, \tilde{c}_2^H, \tilde{y}^H/\omega^H)\\ &= u(\frac{\tilde{y}^H - T^L - \phi(\tilde{y}^H - \hat{y}^L) - (1+t_2)\tilde{c}_2^H}{1+t_1}, \tilde{c}_2^H, \tilde{y}^H/\omega^H)\\ &= u(\tilde{c}_1^H, \frac{\tilde{y}^H - T^L - \phi(\tilde{y}^H - \hat{y}^L) - (1+t_1)\tilde{c}_1^H}{1+t_2}, \tilde{y}^H/\omega^H).\end{aligned}$$

We thus have

$$\frac{\partial \tilde{V}^H}{\partial t_1} = -\frac{\tilde{c}_1^H}{1+t_2}\tilde{u}^h_{c_2}, \quad \frac{\partial \tilde{V}^H}{\partial t_2} = -\frac{\tilde{c}_2^H}{1+t_1}\tilde{u}^h_{c_1} = \frac{\tilde{c}_2^H}{\tilde{c}_1^H}\frac{\partial \tilde{V}^H}{\partial t_1}, \quad \frac{\partial \tilde{V}^H}{\partial \hat{y}^h} = \frac{\phi'(\tilde{y}^H - \hat{y}^L)}{1+t_1}\tilde{u}^h_{c_1}, \quad \frac{\partial \tilde{V}^H}{\partial T^L} = -\frac{1}{1+t_1}\tilde{u}^h_{c_1}. \tag{4}$$

For the same reason as in the previous section, we have $\tilde{y}^H > y^L$, $\tilde{c}_1^H > c_1^L$ and $\tilde{c}_2^H > c_2^L$.

The social planner chooses the tax rates t_1, t_2 and proposes two bundles $(T^L, \hat{y}^L)$ and $(T^H, \hat{y}^H)$. These bundles are designed such that individuals optimally choose the one intended for them. A utilitarian planner thus solves the following program:

$$\max_{t_1,t_2,T^h,\hat{y}^h} n^L V^L(t_1, t_2, \hat{y}^L, T^L) + n^H V(t_1, t_2, \hat{y}^H, T^H)$$

st

$$t_1(n^L c_1^L + n^H c_1^H) + t_2(n^L c_2^L + n^H c_2^H) + n^L T^L + n^H T^H \geq 0$$

$$V^H(t_1, t_2, \hat{y}^H, T^H) \geq V^H(t_1, t_2, \hat{y}^L, T^L).$$

The first-order conditions are:

$$n^L\frac{\partial V^L}{\partial t_1} + n^H\frac{\partial V^H}{\partial t_1} + \mu(n^L c_1^L + n^H c_1^H + t_1(n^L\frac{\partial c_1^L}{\partial t_1} + n^H\frac{\partial c_1^H}{\partial t_1}) + t_2(n^L\frac{\partial c_2^L}{\partial t_1} + n^H\frac{c_2^H}{\partial t_1})) + \lambda(\frac{\partial V^H}{\partial t_1} - \frac{\partial \tilde{V}^H}{\partial t_1}) = 0$$

$$n^L\frac{\partial V^L}{\partial t_2} + n^H\frac{\partial V^H}{\partial t_2} + \mu(n^L c_2^L + n^H c_2^H + t_1(n^L\frac{\partial c_1^L}{\partial t_2} + n^H\frac{\partial c_1^H}{\partial t_2}) + t_2(n^L\frac{\partial c_2^L}{\partial t_2} + n^H\frac{\partial c_2^H}{\partial t_2})) + \lambda(\frac{\partial V^H}{\partial t_2} - \frac{\partial \tilde{V}^H}{\partial t_2}) = 0$$

$$n^L\frac{\partial V^L}{\partial T^L} + \mu(t_1 n^L\frac{\partial c_1^L}{\partial T^L} + t_2 n^L\frac{\partial c_2^L}{\partial T^L} + n^L) + \lambda(-\frac{\partial \tilde{V}^H}{\partial T^L}) = 0$$

$$n^H\frac{\partial V^H}{\partial T^H} + \mu(t_1 n^H\frac{\partial c_1^H}{\partial T^H} + t_2 n^H\frac{\partial c_2^H}{\partial T^H} + n^H) + \lambda\frac{\partial V^H}{\partial T^H} = 0.$$

We use (3) and (4), as well as the Slutsky relationship:

$$\frac{\partial c_j^h}{\partial t_i} = S_{ji}^h + c_i^h\frac{\partial c_j^h}{\partial T^h}.$$

Multiplying the third and fourth conditions by c_1^L and c_1^H respectively and adding up, we obtain:

$$\begin{aligned} n^L c_1^L \frac{\partial V^L}{\partial T^L} + \mu(t_1 n^L c_1^L \frac{\partial c_1^L}{\partial T^L} + t_2 n^L c_1^L \frac{\partial c_2^L}{\partial T^L} + n^L c_1^L) + \lambda c_1^L (-\frac{\partial \tilde{V}^H}{\partial T^L}) & \\ + n^H c_1^H \frac{\partial V^H}{\partial T^H} + \mu(t_1 n^H c_1^H \frac{\partial c_1^H}{\partial T^L} + t_2 n^H c_1^H \frac{\partial c_2^H}{\partial T^H} + n^H c_1^H) + \lambda c_1^H \frac{\partial V^H}{\partial T^H} &= 0. \end{aligned}$$

Substracting from the first condition yields:

$$\frac{\partial \mathcal{L}}{\partial t_1} = \mu(t_1(n^L S_{11}^L + n^H S_{11}^H) + t_2(n^L S_{21}^L + n^H S_{21}^H)) + \lambda \frac{\partial \tilde{V}^H}{\partial T^L}(c_1^L - \tilde{c}_1^H).$$

Similar computations lead to:

$$\frac{\partial \mathcal{L}}{\partial t_2} = \mu(t_1(n^L S_{12}^L + n^H S_{12}^H) + t_2(n^L S_{22}^L + n^H S_{22}^H)) + \lambda \frac{\partial \tilde{V}^H}{\partial T^L}(c_2^L - \tilde{c}_2^H).$$

To assess whether the two tax rates should be differentiated, we fix one of the two to its optimal level, w.l.o.g. good 1, so that $\partial \mathcal{L}/\partial t_1 = 0$ and evaluate:

$$\begin{aligned} \left.\frac{\partial \mathcal{L}}{\partial t_2}\right|_{t_2=t_1\equiv t} &= \mu(t(n^L S_{12}^L + n^H S_{12}^H) + t(n^L S_{22}^L + n^H S_{22}^H)) + \lambda \frac{\partial \tilde{V}^H}{\partial T^L}(c_2^L - \tilde{c}_2^H) \\ &= \mu(t(n^L S_{12}^L + n^H S_{12}^H) + t(n^L S_{22}^L + n^H S_{22}^H)) - \mu(t(n^L S_{11}^L + n^H S_{11}^H) + t(n^L S_{21}^L + n^H S_{21}^H)) \frac{c_2^L - \tilde{c}_2^H}{c_1^L - \tilde{c}_1^H}. \end{aligned}$$

This expression is positive (meaning that $t_2 > t_1$) if and only if:

$$\frac{n^L S_{12}^L + n^H S_{12}^H + n^L S_{22}^L + n^H S_{22}^H}{n^L S_{11}^L + n^H S_{11}^H + n^L S_{21}^L + n^H S_{21}^H} < \frac{c_2^L - \tilde{c}_2^H}{c_1^L - \tilde{c}_1^H}.$$

In order to interpret this formula, we differentiate the budget constraint of the households:

$$(1+t_1)c_1^h + (1+t_2)c_2^h = y^h - T^h - \phi(y^h - \hat{y}^h),$$

to obtain:

$$\begin{aligned} (1+t_1)S_{11}^h + (1+t_2)S_{21}^h &= (1-\phi'^h)S_{y1}^h \\ (1+t_1)S_{12}^h + (1+t_2)S_{22}^h &= (1-\phi'^h)S_{y2}^h \end{aligned}$$

The condition for having different tax rates (t_2 larger than t_1) then becomes:

$$\frac{n^L(1-\phi'^L)S_{y2}^L + n^H(1-\phi'^H)S_{y2}^H}{n^L(1-\phi'^L)S_{y1}^L + n^H(1-\phi'^H)S_{y1}^H} < \frac{c_2^L - \tilde{c}_2^H}{c_1^L - \tilde{c}_1^H}. \quad (5)$$

The left-hand side measures the ratio of the *efficiency* effects of the two tax rates. As explained in the previous section, an increase in a tax rate induces individuals to substitute leisure for consumption, implying a lower income level and thus less tax revenues. The lower this ratio, the more desirable it is to increase t_2 for efficiency purposes. The right-hand side represents the *redistributive* impact of the tax rates. The lower this ratio, the greater the redistributive impact of increasing t_2 relatively to t_1.

Whether t_2 should be greater or lower than t_1 depends on the comparison between the efficiency and the redistributive effect. It is not sufficient to say that t_2 for example should be increased because it has a greater redistributive impact, in the sense that it induces a larger gap between the consumption of the poor and the rich mimicker than t_1. This redistributiveness has to be balanced against the efficiency implications of increasing each of the two tax rates.

4.1 Conditions for a uniform tax structure

It is difficult to find general conditions on the utility function for having tax differentiation or not. A widely studied utility function in tax analysis is the weakly separable utility: $u(c_1, c_2, l) = u(f(c_1, c_2), l)$. With this property, the Atkinson-Stiglitz theorem (Atkinson and Stiglitz (1976)) holds: when labor income can be taxed in a nonlinear way, there is no need to tax goods. Deaton (1979) shows that, if additionally Engel curves are linear, commodity taxation is useless when there is a linear tax on income.

The conditions used by Deaton are also sufficient in our framework for having *uniform* commodity taxation (this was already noted by Boadway et al. (1994)). Under these conditions, we indeed have:

$$c_i^h = \gamma_i(p) + \beta_i(p)x^h,$$

where x^h is the disposable income of a type h individual. Moreover, with a weakly separable utility function (Goldman and Uzawa (1964), Sandmo (1974)):

$$S_{yi}^h = \varphi^h \frac{\partial c_i^h}{\partial x^h} = \varphi^h \beta_i(p).$$

One can easily check that in this case the efficiency and redistributive effects exactly cancel out, so that the optimal taxes on goods are uniform.

4.2 Nonlinear Engel curves

With nonlinear Engel curves, we can distinguish between necessity and luxury goods. Necessities (resp. luxuries) have concave (resp. convex) Engel curves. We show through a simple example that the redistributive and efficiency effects identified above may go in opposite direction and therefore that one cannot conclude about the desirability of taxing more luxuries or commodities.

We consider the following family of Engel curves:

$$c_i^h = \gamma_i (x^h)^{\alpha_i}.$$

We assume w.l.o.g. that $\alpha_1 < 1$ (necessity) and $\alpha_2 > 1$ (luxury). We consider for simplicity a weakly separable utility function, implying that:

$$\begin{aligned} S_{yi}^h &= \varphi^h \frac{\partial c_i^h}{\partial x^h} \\ &= \varphi^h \gamma_i \alpha_i (x^h)^{\alpha_i - 1}. \end{aligned}$$

From (5), we can compute the redistributive and efficiency effects. These are respectively:

$$\frac{c_2^L - \tilde{c}_2^H}{c_1^L - \tilde{c}_1^H} = \frac{\gamma_2((x^L)^{\alpha_2} - (\tilde{x}^H)^{\alpha_2})}{\gamma_1((x^L)^{\alpha_1} - (\tilde{x}^H)^{\alpha_1})}$$

and

$$\begin{aligned} & \frac{n^L(1-\phi'^L)S_{y2}^L + n^H(1-\phi'^H)S_{y2}^H}{n^L(1-\phi'^L)S_{y1}^L + n^H(1-\phi'^H)S_{y1}^H} \\ = \; & \frac{n^L(1-\phi'^L)\varphi^L\gamma_2\alpha_2(x^L)^{\alpha_2-1} + n^H(1-\phi'^H)\varphi^H\gamma_2\alpha_2(x^H)^{\alpha_2-1}}{n^L(1-\phi'^L)\varphi^L\gamma_1\alpha_1(x^L)^{\alpha_1-1} + n^H(1-\phi'^H)\varphi^H\gamma_1\alpha_1(x^H)^{\alpha_1-1}}. \end{aligned}$$

When comparing the two effects, the terms γ_2/γ_1 cancel out on both sides. It thus appears that the redistributive effect calls for taxing more the luxury (good 2). But the efficiency effect goes in the opposite directions: taxing the luxury generates more distortions (this is obtained by noting that the above ratio is larger than 1 with $\alpha_2 > \alpha_1$). Which effect dominates is thus not clear. Some empirical studies are needed to obtain more precise answers.

5 Conclusion

Tax avoidance is increasingly recognized as one of the major problems faced by fiscal administrations in modern democracies. Individuals devote a considerable amount of resources to exploit the loopholes of the tax systems, so as to end up paying as little taxes as possible.

In this article, we have, like Boadway et al. (1994)), analyzed how tax avoidance affects the design of the tax structure. Our main conclusion is that is provides a rationale for taxing consumption goods, as it is a way to tax true income.

We have also shed some light on the issue of tax differentiation. Whether luxuries or necessities should me taxed more depend on the comparison of a redistributive and an efficiency effects. It is difficult to determine analytically which of the two effects dominates. This work should thus be complemented with a careful empirical analysis.

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